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Evaluation report of the Ph.D. thesis

'Constitutive Modelling of Composites with Elastomer Matrix and Fibres with Significant Bending Stiffness'

by Svitlana Fedorova

Description of the thesis

The thesis includes an extended list of nomenclature, an introduction, eight main chapters, a summary and one appendix. The list of references provides an overview on literature relevant to the research topic of the thesis.

The thesis deals with the modelling and simulation of composites with elastic matrix and elastic fibres – with inextensible fibres included as a special case, respectively constraint. Inelastic effects, such as visco-elastic or elasto-plastic response of the matrix, is not addressed in the thesis. The main focus is the incorporation of bending contributions of fibres. The main theoretical modelling background dates back to a publication by Spencer and Soldataos from 2007. Numerical examples provided in the thesis are based on the algorithmic formulation established in the thesis by Lasota published in 2013.

Svitlana Fedorova develops new models and motivates models and simulation results with experiments, respectively measured data. Moreover, analytical solutions are newly developed and discussed. The general framework is established for the large strain setting, whereas most (analytical) examples refer to the small strain case.

Chapter 1 introduces the reader to the research field of the thesis and provides an overview on the goals, content and structure of the thesis.

Chapter 2 provides a discussion on the state of the art of the modelling and simulation of composites. The model subsequently discussed in the thesis includes second gradients of the motion, i.e. the gradient of the deformation gradient, and has originally been introduced within the framework of a Mindlin theory. In this context Svitlana Fedorova provides a general overview and classification of higher-grade and higher-order theories.

Chapter 3 focuses on experimental investigations and basic modelling of fibre-reinforced composites. The experimental tests performed include tension loading as well as bending deformation, whereby the specimens considered included different (but homogeneously distributed) fibre orientations. The experimental results show inelastic effects, such as the Mullins effect, whereas the model is referred to solely elastic deformations. The model used is well-established in the literature, namely finite elasticity in combination with one structural (rank one) tensor so that the stretch in fibre direction can be included as additional argument of the strain energy function. This basic model cannot fully capture the response under bending deformation as experimentally observed, which motivates to further extend the model by additional bending contributions of the fibres.

Chapter 4 discusses assumptions of basic mixture rules in the context of composites. To be specific, the combination of the Voigt assumption (in fibre direction) and the Reuss assumption (transverse with respect to fibre direction) is addressed. The restriction to small strain kinematics is used. When additionally accounting for fibre bending, representation of the resulting stress and couple stress contributions are highlighted.

Chapter 5 further extends the content of Chapter 4 to the large strain setting. The discussion is based on the model proposed by Spencer and Soldatos in 2007. The resulting invariants are highlighted – under the assumption that the gradient of the deformation gradient is considered in form of its projection onto the fibre direction only. The constitutive model used is summarised with (as common) the volumetric part of the couple stress tensor remaining undetermined. A relation between the invariants used and the bending mode of fibres is shown. Thereafter, the material parameters introduced are related to specific states of deformation such as pure bending.

Moreover, the model is implemented in a finite element framework based on the formulation established in the thesis by Lasota published in 2013. This, together with subsequent discussions on analytical solutions for the small strain case, can be considered as the main contribution of the thesis by Svitlana Fedorova. In order to include higher (second) gradients of the displacement field in the computational model, Hermite polynomials are used to approximate the displacement field and related test functions. Based on this, several examples are discussed.

Chapter 6 develops analytical solutions of a plane disc within the framework of polar elasticity under the assumption of small strain kinematics. The results are based on the work by Farhat and Soldatos published in 2015.

Chapter 7 discusses and compares the response of different models in the framework of polar elasticity at small strains. The deformation mode considered refers to a thick plate under bending.

Chapter 8 addresses the verification of the finite element formulation with analytical solutions, whereby small strain kinematics are assumed. The particular boundary value problems considered for the comparison refer to a beam-like structure. The analysis does not include studies on the influence of the discretisation, respectively number of elements used. The finite element based simulation results turn out to be in good agreement with the analytical solution, whereby the criterion for the comparison is a displacement component at the top surface of the specimen and the distribution of first principal stress, as well as a component of the couple stress tensor both in the mid cross-section of the specimen.

Chapter 9 further extends Chapter 8 by discussing the incorporation of additional constraints such as incompressibility and inextensibility of fibres.

Chapter 10 summarises the content of the thesis and provides perspectives for future research work.

Evaluation of the thesis

The modelling and simulation methods used and developed in the thesis are based on (amongst others) the works by Spencer and Soldatos (2007) and Lasota (2013). The thesis by Svitlana Fedorova includes substantial extensions of these works in view of both, modelling and simulation. Moreover, the thesis includes experimental investigations in order to motivated the models proposed. The general finite element formulation adopted and extended in this work deals with finite deformation (polar) elasticity which, in general, constitutes a highly nonlinear and demanding problem to be solved iteratively. The references on which the thesis is based are clearly mentioned, and the thesis includes new contributions, when compared to models and finite element applications previously established in the literature.

Svitlana Fedorova shows that she has command of both the development of new constitutive models as well as their implementation. She runs advanced finite element simulations and discusses the simulation results in detail and in the context of comparison with analytical models, the solutions of which constitute a main (theoretical) contribution of the thesis. The research work of Svitlana Fedorova is interdisciplinary at the intersection of engineering science and applied mathematics and at the international level of Ph.D. work in this research area.

In conclusion, Svitlana Fedorova shows that she is competent to carry out scientific work and to apply scientific methods to new and interdisciplinary research topics. I recommend to continue the Ph.D. procedure and advocate the thesis by Svitlana Fedorova for defence.

Questions related to the thesis

- Which modelling framework would be suitable to include inelastic behaviour, such as visco-elasticity or elasto-plasticity, for the matrix material?
- Which modelling framework would be suitable to include debonding effects between fibres and the matrix material?
- How can initial curvature of fibres, i.e. non-straight fibres in the undeformed state, be included in the modelling framework?
- The Finite Element Formulation developed is based on the usage of Hermite polynomials in order to include second derivatives of the shape functions, respectively degrees of freedom. Which alternative Finite Element formulations and frameworks can be used? What are the respective advantages and disadvantages of the different approaches?
- Can the finite deformation and invariant-based framework used be extended to the modelling of composites with two fibre families?

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