

**RESPONSE**  
of the official opponent for the dissertation work

Mgr. Hanna Demchenko  
**OPTIMIZATION OF DELAYED DIFFERENTIAL SYSTEMS  
BY LYAPUNOV'S DIRECT METHOD**

**Relevance of the topic of the dissertation.**

The dissertation is devoted to the development of existing methods, namely Malkin's approach and Lyapunov's method, for the synthesis of stabilizing control laws for dynamic systems, which are described by ordinary differential and functional differentiation equations (equations with time delay argument). It should be noted that the tasks of stabilization of movement in control systems with elements of delay are the basis of mathematical research, which are carried out in the design of new transport, space, technical and economic processes and objects. Consequently, the subject of the dissertation is undoubtedly important and relevant.

**Review of the content and main results of the work.**

The Doctoral Thesis present at English and have **89** pages of main text. It consists of an introduction, two chapters, conclusions and a list of used literature.

At the ***Introduction*** author formulate the aim of the thesis: solves a problem of optimal stabilization for differential non-delayed and delayed equations and their systems. She gave a short release current state of the problem – qualitative theory of functional differential equations, theory of stability, control theory and optimization. At the Preliminaries we can see main definitions, statements and theorems related to the stability of functional differential equations and direct Lyapunov method.

At the ***Second chapter*** author investigates a stabilization problem for a system of differential equations without delay argument. She clearly formulates the problem and the main existing Malkin's result. In the section 2.3 we can see some results that continue the Malkin's approach. The Th.2.3.1 – Th.2.3.3 has statements about stabilization and concrete form of optimal control function for non-delay linear scalar process, vector process with scalar control and vector process with vector control, accordingly. All results are supported by examples.

The ***Third chapter*** is devoted the optimization problem in delayed case. The problem is formulated in section 3.1. In section 3.2 proved Th.3.2.1 which contains the generalization of Malkin's results on such case of dynamical systems. Section 3.3 consist from some examples that illustrates the condition of the Th.3.2.1. Last section of the chapter 3 is devoted to an application previous obtaining results on some concrete linear equation and systems with time-delay

arguments.

**Scientific novelty and practical significance of the results of the dissertation.**

Among the main scientific results of the dissertation Hanna Demchenko can be distinguished the following:

- In chapter 2 author proved three theorems, which are a good application of well-known Malkin's theorem for some classes of linear dynamical system without delay, and supported them with examples.
- Chapter 3 contains the key result of the dissertation – this is the Th.3.2.1. This theorem formally is a generalization of Malkin's approach on case of functional differential systems with time-delay argument. With using this theorem and its corollaries (Th.3.4.1, 3.4.4, 3.4.7, 3.4.7, 3.4.13, 3.14.15) the author shows successful applications to linear delayed equations and systems.
- It should be particularly noted the result of section 3.4.6 – Th.3.4.15, where is investigated the system with multiple constant delays.

**The validity and reliability of the main assertions and conclusions** is confirmed by the rigorous proofs of the results on the basis of the logical use of known facts on the theory of stability, optimal control theory and matrix analysis, as well as conducting analytical numerical examples.

The Short Version of PhD Thesis correctly reflects the basic maintenance of dissertation work. The main results of the dissertation are published in **2** articles in professional scientific journals, **9** papers of conference proceedings and **10** abstracts of scientific conference. Some of them includes in the Web of Science. They are sufficiently tested at seminars and international conferences. The dissertation and its short version are executed according to the established requirements.

**Remark and questions.**

1) Part 1 **Introduction**, on my opinion, is so short and not complete for posed such well-known scientific problems. The work will be much more presentable, if author point the references on (as the minimum):

- a) works devoted to the systems with deviating argument – Kolmanovskii V.B., Nosov V.R., Datko R.
- b) works devoted to the Lyapunov's method – Chetaev N.G., Martynyuk A.A.
- c) works devoted to the stabilization problem – Mazko A.G., Polyak B.T., Doyle J.C.

2) There are some mistakes or misprints in the text of the dissertation. For example,

- a) At p.19 system (2.1) was called as “functional differential” system, but it is ordinary differential system
- b) At p.20 Problem 2.1.1 was formulated as “optimal control problem”, but in fact, it is the “optimal stabilization problem”

- 3) Part 2 will be more good and complete, if author can compare the results obtained in dissertation with known similar results, which were derived by using other methods (dynamical programming Bellman's method, maximum principle of Pontryagin, etc.).
- 4) There are many examples in the dissertation work, which itself are very good, but all of them are so-called an "academic" examples. Please, can you present an example that describes a real process in engineering sciences, economy, ecology, etc. This fact will underline importance of results, obtained in dissertation.
- 5) All results obtaining in dissertation work deal with continuous dynamical systems. But in real life many processes are discrete and are described in terms of difference equations. Please, can you explain your results to this case, and if "yes", show a brief scheme of application.

The above remarks are not essentially critical, do not affect the positive assessment of the dissertation and do not reduce its theoretical and practical significance. In general, the work is presented consistently and correctly.

### **Conclusions.**

On the basis of the above, I think that the dissertation work Hanna Demchenko "Optimization of Delayed Differential Systems by Lyapunov's Direct Method" is a high-level scientific work that meets the all requirements put forward for the Ph.D theses in the area of Mathematics in Electrical Engineering. The candidate has shown the breadth of scientific thinking, high erudition, and the ability to work independently. Therefore, I believe that the author deserves the award of the scientific degree of Ph.D.

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Doc. Andriy Shatyrko, CSc.  
Department of Complex Systems Modelling,  
Faculty of Computer Science and Cybernetics,  
Taras Shevchenko National University of Kyiv,  
Ukraine